THE EXCITATION OF THE CHANDLER WOBBLE

by

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Abstract. Any irregularly shaped solid body rotating about some axis that is not aligned with its figure axis will wobble as it rotates. For the Earth, this Eulerian free wobble is known as the Chandler wobble in honor of S. C. Chandler who first observed it in 1891[1]. From the observations of the Chandler wobble taken since its discovery, its period has been estimated to be 433.0 ± 1.1 (1σ) days and its *e*-folding amplitude decay time has been estimated to be 68 years (with 1σ bounds of 28 and 298 years)[2]. Because a damping time of 68 years is short on a geological time-scale, the amplitude of the Chandler wobble should quickly dampen to zero unless some mechanism or combination of mechanisms are exciting it. Since its discovery, many mechanisms have been evaluated, without success, to determine whether or not they could be the excitation mechanism(s) of the Chandler wobble including atmospheric processes[3–7], continental water storage[8–11], core-mantle interactions[12–16], and earthquakes[17,18]. Here, evidence is presented that the Chandler wobble is excited by a combination of atmospheric and oceanic processes, with ocean-bottom pressure fluctuations being the dominant excitation mechanism.

Measurements of the Earth's changing rotation are currently made by the space-geodetic techniques of satellite and lunar laser ranging, very long baseline interferometry, and global positioning system interferometry[19]. The Earth rotation series used in this study is a combination of these space-geodetic measurements known as SPACE97[20] and consists of daily averaged values of Universal Time, polar motion, and their rates of change spanning 1976.7–1998.0. Strictly speaking, the polar motion parameters specify the location of the Celestial Ephemeris Pole (CEP) within the body-fixed terrestrial reference frame and will be so interpreted here. However, for periods long compared to a day, such as for the Chandler wobble, and to sufficient accuracy, the polar motion parameters can be interpreted as specifying the location of the rotation pole within the terrestrial reference frame[21–23].

Polar motion consists largely of: (1) a forced annual wobble having a nearly constant amplitude of about 100 milliarcseconds (mas), (2) the free Chandler wobble having a variable amplitude ranging between about 100 to 200 mas, (3) quasi-periodic variations on decadal time scales having amplitudes of about 30 mas known as the Markowitz wobble, (4) a linear trend having a rate of about 3.5 mas/yr, and (5) smaller amplitude variations occurring on all measurable time scales. This rich polar motion spectrum is caused by the rich variety of processes forcing polar motion. In the absence of external torques, the polar motion parameters x_p and y_p can be related to the processes forcing polar motion by linearizing the Liouville equation which expresses the conservation of angular momentum within a rotating, body-fixed reference frame[19,24]:

$$\mathbf{p}(t) + \frac{i}{\sigma_{cw}} \frac{d\mathbf{p}(t)}{dt} = \chi(t) \tag{1}$$

where σ_{cw} is the complex-valued frequency of the Chandler wobble and the complex-valued quantity $\mathbf{p} \equiv (x_p - i\, y_p)$ specifies the x- and y-coordinates, x_p and y_p respectively, of the CEP with x_p being positive towards the Greenwich meridian and y_p being positive by convention towards 90°W longitude. Equation 1 is the expression for simple harmonic motion in the complex plane with the right-hand-side $\chi(t)$ being the forcing, or excitation, function which can be written as[25]:

$$\chi(t) = \frac{1.61}{\Omega\,(C\!-\!A)} \left[\mathbf{h}(t) + \frac{\Omega\, \mathbf{c}(t)}{1.44} \right] \tag{2}$$

where C and A are the greatest and least, respectively, principal moments of inertia of the Earth, the mean angular velocity of the Earth is Ω, the complex-valued quantity $\mathbf{c}(t) \equiv c_{13}(t) + i\; c_{23}(t)$ represents changes in the two indicated elements of the Earth's inertia tensor such as those due to atmospheric or oceanic mass redistribution, and the complex-valued quantity $\mathbf{h}(t) \equiv h_1(t) + i\, h_2(t)$ represents relative angular momentum changes such as those due to changes in the atmospheric winds or oceanic currents. The factor of 1.61 includes the effect of core decoupling and the factor of 1.44 in the denominator of the second term in the square brackets in Equation 2 accounts for the yielding of the solid Earth due to its changing load.

Because polar motion is resonant at the Chandler frequency (Equation 1), investigations of the excitation of the Chandler wobble are usually conducted by frequency-domain comparisons of the observed polar motion excitation functions with those computed from various geophysical processes. Here, the observed polar motion excitation functions are those determined from the SPACE97 polar motion values and rates using Equation 1. They are then compared to the excitation functions caused by atmospheric wind and pressure changes and by oceanic current and ocean-bottom pressure changes. The atmospheric excitation functions used here are those computed from the National Centers for Environmental Prediction (NCEP) / National Center for Atmospheric Research (NCAR) reanalysis project[26] and are available from the International Earth Rotation Service (IERS) Special Bureau for the Atmosphere[27]. The oceanic excitation functions used here are those computed by Ponte *et al.*[28] and are available from the IERS Special Bureau for the Oceans.

Ponte *et al.*[28] computed the polar motion excitation functions due to oceanic currents and, separately, ocean-bottom pressure changes from the products of a global oceanic general circulation model (OGCM) driven by 12-hour wind stress fields and daily surface heat and fresh water flux fields from NCEP. Atmospheric pressure was not used to force the OGCM. The ocean-bottom pressure excitation term was corrected by them for the effects of volume changes due to steric effects within the Boussinesq OGCM by adding a uniform sea level layer of fluctuating thickness to the sea surface height fields produced by the OGCM. The resulting oceanic current and ocean-bottom pressure excitation functions are 5-day-averaged values spanning January 1985 to April 1996.

The available atmospheric wind and pressure excitation functions computed from the NCEP/NCAR reanalysis project are 6-hour values spanning 1958.0 to the present. The pressure excitation term is available under two different assumptions for the response of the oceans to surface pressure changes: (1) the inverted barometer assumption wherein the oceans are assumed to respond isostatically to the imposed surface pressure variations, and (2) the rigid ocean assumption wherein the oceans are assumed to fully transmit without delay or attenuation the atmospheric pressure fluctuations to the ocean-bottom. Since at periods long compared to a day the inverted barometer approximation should be valid[29], the pressure term computed under this assumption has been used here.

Since the oceanic current and ocean-bottom pressure excitation functions are given as 5-day-averaged values, 5-day-averaged values were also formed of the daily averaged observed excitation functions and of the 6 hourly atmospheric wind and pressure excitation functions. In order to reduce spectral leakage into the Chandler frequency band of annual excitation processes, a seasonal signal was removed from the 5-day-averaged observed, atmospheric, and oceanic excitation functions by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. This fit was done on that subset of the series spanning 1985.0–1996.0 in order to fit an integral number of annual and semiannual oscillations. All subsequent analysis will be done on this 11 year subset of the residual excitation series.

Figure 1 shows power spectral density (psd) estimates of the observed (black curve), atmospheric (red curve), and sum of atmospheric and oceanic (green curve) excitation functions from which seasonal signals have been removed. A Hanning window was applied to each series prior to forming the spectral estimates. In agreement with the conclusion of previous studies[3,4], it is seen that the sum of atmospheric wind and pressure fluctuations (red curve) does not have sufficient power to excite the Chandler wobble (the Chandler frequency of 0.8435 cycles/year (cpy) is indicated by the vertical dotted line). However, a good match to the observed Chandler wobble excitation power is obtained upon adding the excitation due to oceanic current and ocean-bottom pressure fluctuations to that due to atmospheric wind and pressure variations (green curve).

Since the time series whose spectra are displayed in Figure 1 consist of 800 5-day-averaged samples, the frequency resolution of these time series is 0.0913 cpy which is just sufficient to resolve the Chandler frequency band. Near the Chandler frequency, the spectral estimates shown in

Figure 1 are given at frequencies of 0.730 cpy, 0.822 cpy, and 0.913 cpy. Integrating the power spectral density estimates of Figure 1 across the Chandler frequency band, taken here to range between 0.730 cpy and 0.913 cpy, gives the power in the Chandler band shown in Table 1 for the various excitation mechanisms being studied here. The observed excitation power in this band is 4.97 mas^2 with the sum of the power due to atmospheric wind, atmospheric pressure, oceanic current, and ocean-bottom pressure excitation being slightly more than this at 5.44 mas^2. Ocean-bottom pressure fluctuations are seen to be the single most important mechanism exciting the Chandler wobble, containing about twice as much power in the Chandler band as that due to atmospheric pressure fluctuations. Oceanic current and atmospheric wind variations are minor contributors to the Chandler wobble excitation, having power of only 0.12 mas^2 and 0.32 mas^2, respectively, in the Chandler band. Destructive interference between the individual excitation processes and statistical fluctuation probably account for the discrepancy between the sum of the individual excitation power estimates and the total atmospheric and/or oceanic excitation power estimates given in Table 1. An example of such destructive interference can be seen by comparing the excitation power in the Chandler band due to the sum of atmospheric and ocean-bottom pressure changes to that obtained when additionally including excitation due to atmospheric wind and oceanic current changes. The destructive interference between the atmospheric wind and oceanic current excitation with the atmospheric and ocean-bottom pressure excitation reduces the power in the Chandler band from 6.28 mas^2 to 5.44 mas^2.

Figure 2 shows the magnitude of the squared-coherence between the observed excitation functions and those due to atmospheric wind and pressure variations (red curve), the sum of ocean-bottom pressure and atmospheric pressure fluctuations (blue curve), and the total sum of atmospheric wind, atmospheric pressure, oceanic currents, and ocean-bottom pressure variations (green curve). The squared-coherence estimates were obtained by averaging over 5 frequency intervals and the 95% and 99% confidence limits on the magnitude of the squared-coherence estimates are indicated by the horizontal dashed lines. As can be seen, near the Chandler frequency (indicated by the vertical dotted line) atmospheric wind and pressure excitation is not coherent with the observed excitation, but that due to the sum of atmospheric wind, atmospheric pressure, oceanic currents, and ocean-bottom pressure is coherent with greater than 99% confidence. Since the sum of atmospheric pressure and ocean-bottom pressure is also coherent with the observed excitation near the Chandler frequency, the addition of atmospheric wind and oceanic current excitation reduces the power in the Chandler band to nearly that observed, but does not affect the coherence.

Numerous investigations have been conducted during the past century in attempts to elucidate the excitation mechanism of the Chandler wobble. Here it has been shown that during 1985.0–1996.0 the single most important mechanism exciting the Chandler wobble has been ocean-bottom pressure fluctuations, which contribute about twice as much excitation power in the Chandler frequency band as do atmospheric pressure fluctuations. Atmospheric winds and oceanic currents have been shown here to play only a minor role in exciting the Chandler wobble during this time. The ability to elucidate the role of atmospheric and ocean-bottom pressure fluctuations in exciting the Chandler wobble is a testament to the fidelity of the atmospheric and oceanic general circulation models that were used to compute the atmospheric and oceanic angular momentum estimates used in this study. The wide distribution of these atmospheric and oceanic angular momentum estimates by the IERS Special Bureaus for the Atmosphere and Oceans enables the type of interdisciplinary research whose results are reported here.

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Table 1 Chandler band excitation power

Excitation process	Power, mas^2
Observed	4.97
Atmospheric	
wind	0.32
pressure (i.b.)	1.87
wind plus pressure (i.b.)	1.44
Oceanic	
currents	0.12
ocean-bottom pressure	3.45
currents plus ocean-bottom pressure	3.69
Atmospheric plus oceanic	
wind plus currents	0.67
i.b. plus ocean-bottom pressure	6.28
Total of all atmospheric plus oceanic	5.44

i.b., inverted barometer

Figure 1. Power spectral density (psd) estimates in decibels (db) computed from time series of polar motion excitation functions $\chi(t)$ spanning 1985.0–1996.0 of: (a) the observed SPACE97 polar motion excitation function derived from space-geodetic Earth rotation measurements (black curve), (b) the sum of the excitation functions due to atmospheric wind and pressure changes (red curve) where the atmospheric pressure term is that computed assuming the inverted barometer approximation is valid, and (c) the sum of all atmospheric and oceanic excitation processes being studied here, namely, the sum of the excitation functions due to atmospheric winds, atmospheric pressure (inverted barometer), oceanic currents, and ocean-bottom pressure (green curve). A seasonal signal has been removed from all series prior to spectral estimation by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. The vertical dotted line indicates the Chandler frequency of 0.8435 cycles/year (cpy). The retrograde component of polar motion excitation is represented by negative frequencies, the prograde component by positive frequencies. The Chandler wobble is a strictly prograde oscillation.

Figure 2. The magnitude of the squared-coherence between the observed polar motion excitation functions spanning 1985.0–1996.0 and the excitation functions due to: (a) the sum of atmospheric wind and pressure changes (red curve) where the pressure term is that computed under the inverted barometer approximation, (b) the sum of atmospheric (inverted barometer) and ocean-bottom pressure fluctuations (blue curve), and (c) the sum of all the atmospheric and oceanic excitation processes being studied here, namely, the sum of atmospheric wind, atmospheric pressure (inverted barometer), oceanic current, and ocean-bottom pressure variations. A seasonal signal has been removed from all series prior to coherence estimation by least-squares fitting and removing a mean, a trend, and periodic terms at the annual and semiannual frequencies. The vertical dotted line indicates the Chandler frequency of 0.8435 cycles/year (cpy) and the horizontal dashed lines indicate the 95% and 99% confidence levels of the magnitude of the squared-coherence.

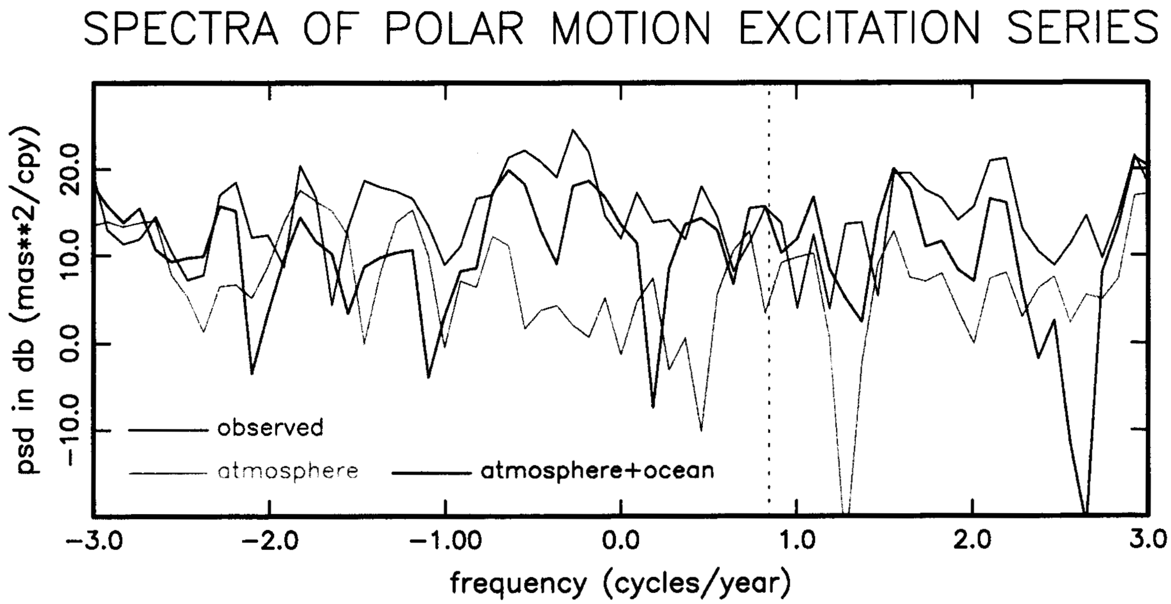
SPECTRA OF POLAR MOTION EXCITATION SERIES
psd in db (mas**2/cpy)
20.0
10.0
0.0
−10.0
observed
atmosphere
atmosphere+ocean
−3.0
−2.0
−1.00
0.0
1.0
2.0
3.0
frequency (cycles/year)

Figure 1

COHERENCE OF OBSERVED AND MODELED EXCITATION

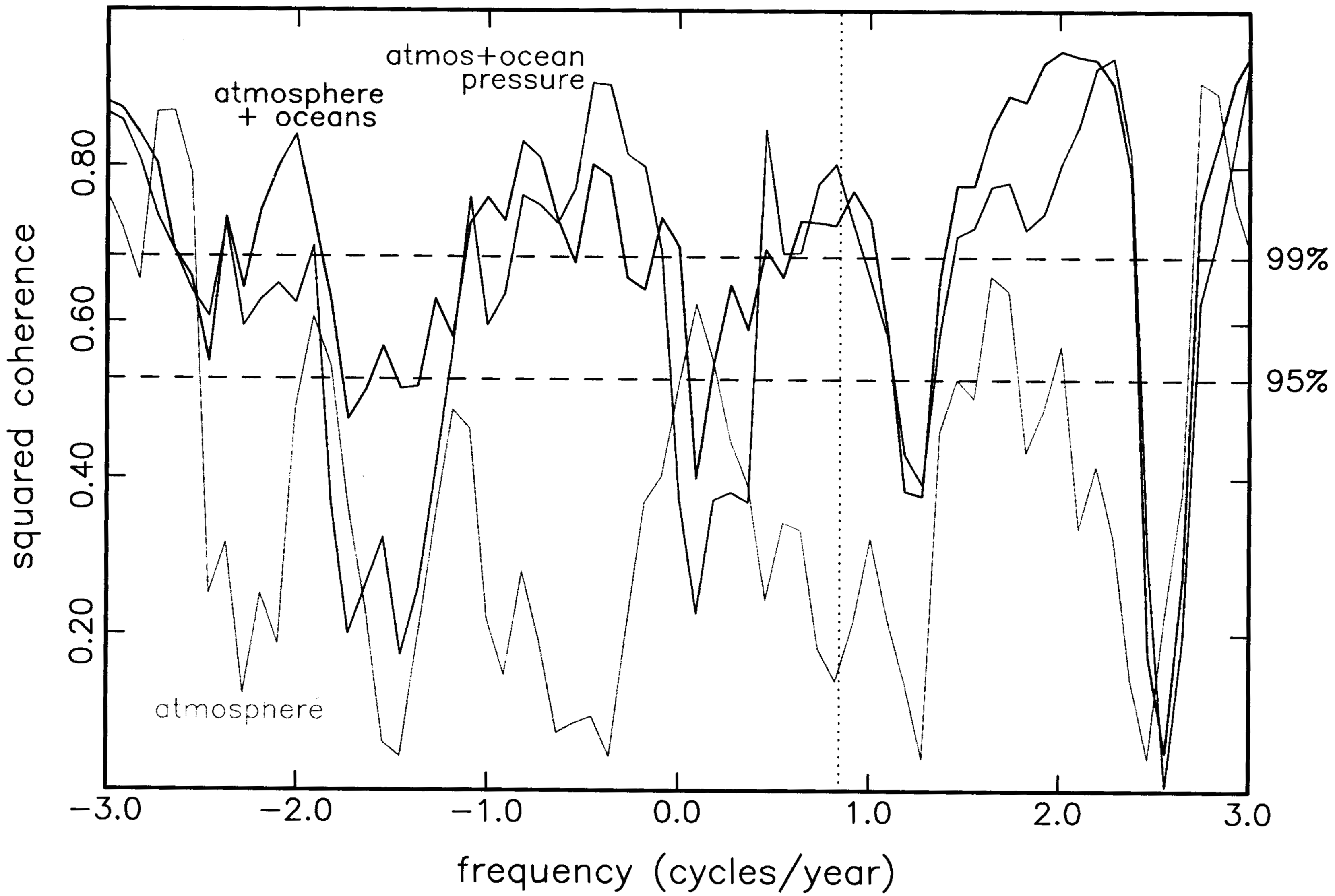

Fig. 2